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Jon V. Pepper

HARSANYI, JOHN CHARLES (*b.* Budapest, Hungary, 29 May 1920; *d.* Berkeley, California, 9 August 2000), *economics, game theory.*

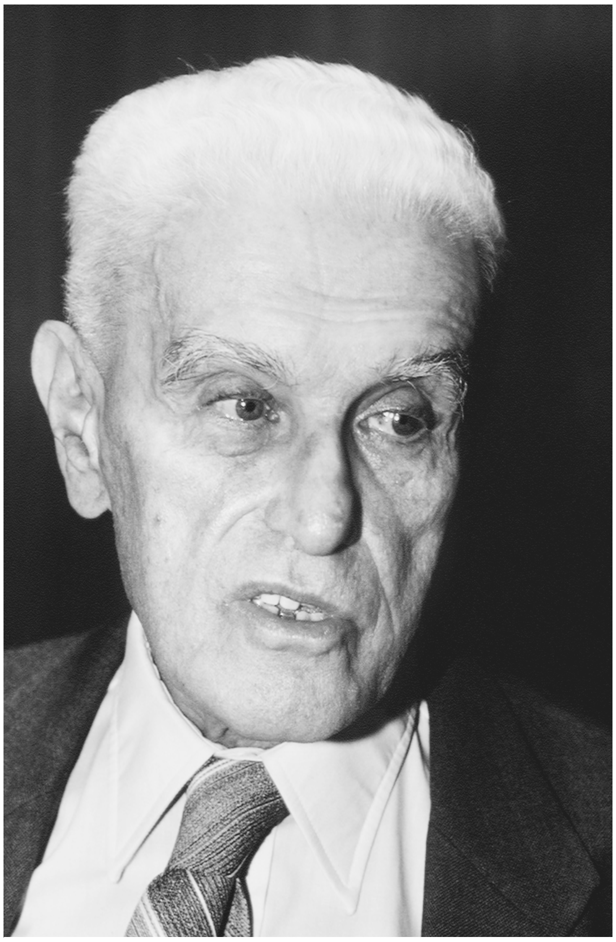
Harsanyi is best known for providing a decision-theoretic foundation for utilitarianism, for his work on equilibrium selection in noncooperative games, and for developing the conceptual foundations for analyzing games of incomplete information. For the latter research, Harsanyi was awarded the Nobel Prize in Economics in 1994 jointly with John Nash and Reinhard Selten.

Early Life and Education. Harsanyi (born Harsányi János Károly) was the only child of Charles and Alice Gombos Harsanyi. His father, a pharmacist by profession, and mother both converted to Catholicism from Judaism. Harsanyi attended the Lutheran Gymnasium in Budapest, whose graduating class of 1921 included one of the founding fathers of game theory, John von Neumann. In 1937, the year of his graduation, Harsanyi won first prize in the national competition for high school students in mathematics. The next two years were spent working in his father's pharmacy.

Although Harsanyi's own inclination was to study mathematics and philosophy, at his father's urging he went to France in 1939 with the intention of enrolling as a chemical engineering student at the University of Lyons. However, having completed a summer course to improve his French in Grenoble, with the outbreak of World War II his parents summoned Harsanyi back to Budapest, where he studied pharmacology, receiving the diploma in pharmacology from the University of Budapest in 1944. By studying pharmacology, Harsanyi received a military deferment that, because of his Jewish background, would have required that he serve in a forced labor unit. With the Nazi occupation of Hungary, Harsanyi lost this exemption and spent seven months doing forced labor in 1944. When his unit was being deported to work in a mine in Yugoslavia, Harsanyi managed to escape at the Budapest railway station. He found sanctuary in a Jesuit monastery until the end of the Nazi occupation. His mother, an asthmatic whose health deteriorated because of the privations of the war, died later that year.

Following World War II, Harsanyi, then a devout Catholic, studied theology (in Latin) in a Dominican seminary, later joining the Dominicans' lay order. However, he lost his faith in his late twenties and was antireligious for the rest of his life. While at the seminary, Harsanyi simultaneously pursued graduate studies at the University of Budapest, to which he returned in 1946. The following year, after completing his dissertation, "The Logical Structure of Errors in Philosophical Arguments," he was awarded a DrPhil, with minors in sociology and psychology.

Harsanyi spent the academic year 1947–1948 as a faculty member of the university's Institute of Sociology, where he met his future wife, Anne Klauber, who was a student in one of his classes. Forced to resign this position because of his anti-Marxist views, Harsanyi spent the next two years running the family pharmacy, which he now co-owned. In April 1950, when confiscation of the pharmacy by the Communist government was imminent, Harsanyi, his future wife, and her parents escaped to Vienna. At the end of that year, they all immigrated to Sydney, Australia, where Anne and John Harsanyi were married in January 1951, a few days after their arrival. Harsanyi became an Australian citizen in 1956. His father was kept on as a poorly paid state employee after his pharmacy was confiscated and subsequently died of kidney failure in 1954.



John Charles Harsanyi. **© FORDEN, PATRICK J. /SYGMA/CORBIS.**

In 1951 Harsanyi enrolled as an evening student in economics at the University of Sydney while spending his days working in a series of factory and clerical jobs. He completed his master of arts degree in economics in late 1953 with a dissertation, "Invention and Economic Growth," and then spent two and a half years as a lecturer at the University of Queensland.

Harsanyi then went to Stanford University on a one-year Rockefeller Fellowship in 1956, where he wrote a game theory doctoral dissertation, "A Bargaining Model for the Cooperative *n*-Person Game," supervised by Kenneth Arrow, a 1972 Nobel laureate. Harsanyi's visa permitted him to stay one more year in the United States, which he did, first spending a semester visiting the Cowles Foundation for Research in Economics at Yale University before returning to Stanford as a visiting assistant professor of economics. In 1958 Harsanyi took up a position as a research fellow at the Australian National University a few months before receiving his PhD in economics from Stanford in 1959.

American Career and Later Life. Feeling isolated because of his colleagues' lack of interest in game theory, Harsanyi returned to the United States where, except for visiting positions, he spent the rest of his career, becoming a U.S. citizen in 1990. From 1961 to 1963, he was a professor of economics at Wayne State University in Detroit. Following a year as a visiting professor at the University of California at Berkeley, Harsanyi became a professor of business administration there in 1965, with a secondary appointment as a professor of economics in 1966. In the years from 1966 to 1968, Harsanyi, together with other prominent game theorists, served as consultants to the U.S. Arms Control and Disarmament Agency under contract to Mathematica, the Princeton-based consulting group that included the game theorists Harold Kuhn and Oskar Morgenstern as principals. Harsanyi retired from Berkeley in 1990.

In addition to his Nobel Prize, Harsanyi's many honors included fellowships in the Econometric Society (1968), the American Academy of Arts and Sciences (1984), and the European Academy of Arts, Sciences, and Humanities (1996), as well as a number of honorary doctorates. He was made a Distinguished Fellow of the American Economic Association in 1994 and an honorary member of the Hungarian Academy of Sciences in 1995. Harsanyi was president of the Society for Social Choice and Welfare in 1996–1997. Harsányi János College in Budapest is named after him.

The Harsanyis had one child, a son, Tom, born in 1964 shortly after their arrival in Berkeley. For some time prior to his death in 2000 from a heart attack, Harsanyi had been in poor health, suffering from Alzheimer's disease.

Foundations of Utilitarianism. For utilitarianism to be a well-defined doctrine, individual well-being must be measurable by a cardinal utility function that permits interpersonal comparisons of utility gains and losses. A function is cardinal if any property of this function that is preserved by multiplying the function by an arbitrary positive constant and then adding a second arbitrary constant is meaningful, as is the case with the scales used to measure temperature. Following the ordinalist revolution of the 1930s, it was thought that no cardinal measure of well-being exists. However, in *Theory of Games and Economic Behavior* (1944), John von Neumann and Oskar Morgenstern argued that the preferences of a rational individual evaluating risky alternatives should conform to a set of properties (axioms) that result in these alternatives being ranked by the expected value of a cardinal utility function, what Harsanyi called Bayesian rationality. Subsequent commentators denied that this utility function had any significance for social welfare analysis.

In his first publication, "Cardinal Utility in Welfare Economics and in the Theory of Risk-Taking" (1953), written while still a student in Sydney, Harsanyi set out to refute this claim. For Harsanyi, welfare judgments are the impersonal preferences expressed by an impartial observer who orders social alternatives based on a sympathetic but impartial concern for the interests of everyone in society. Specifically, the impartial observer engages in a thought experiment in which he imagines having an equal chance of being anyone in society, complete with that person's preferences and objective circumstances. Thus, ranking social alternatives is reduced to a problem in individual decision making under risk and therefore, by applying the von Neumann–Morgenstern expected utility theory, Harsanyi argued that different social states should be ranked by the average of the utilities of all the individuals in society, thereby providing a Bayesian decision-theoretic foundation for average utilitarianism.

The hypothetical choice situation utilized in Harsanyi's impartial observer theorem is an example of what the philosopher John Rawls, in his monograph, *A Theory of Justice* (1971), has called an original position. The idea of deriving substantive principles of morality based on rational individual decision making behind a veil of ignorance (to use another Rawlsian expression), in which morally irrelevant information has been withheld, is arguably Harsanyi's most important contribution to ethics. In Rawls's formulation of this idea, less information is permitted behind the veil, with the consequence, or so Rawls argued, that social institutions should be designed so as to maximize the prospects of the worst-off individuals (once priority has been given to ensuring that everyone enjoys equal liberties and fair equality of opportunity). In "Can the Maximin Principle Serve as a Basis for Morality? A Critique of John Rawls's Theory" (1975), Harsanyi defended his Bayesian use of expected utility theory and argued that Rawls's maximin reasoning leads to unsatisfactory outcomes.

Harsanyi's impartial observer must be able to make interpersonal comparisons of utility gains and losses in order to rank the social lotteries he is faced with. In "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility" (1955), Harsanyi investigated the logical basis for these comparisons. For him, interpersonal utility comparisons are made by empathetic identification; the observer evaluates how well off someone else is in a particular situation by asking how well off he would be if he were put in the place of that individual, complete with that individual's tastes and values. In effect, all interpersonal utility comparisons are reduced to intrapersonal comparisons. Furthermore, these comparisons are empirical statements made on the basis of an a priori principle, Harsanyi's similarity principle, which says that the utility obtained from an alternative by any individual is determined by a function (common to everyone) of the biological and cultural variables that determine tastes and values.

In his 1955 article, Harsanyi also provided an alternative justification for a weighted form of utilitarianism, his social aggregation theorem. In this theorem, alternatives are risky alternatives and all preferences, both individual and social, are assumed to satisfy the von Neumann–Morgenstern expected utility axioms. The individual and social preferences are related to each other by the requirement that if everyone is indifferent between two alternatives, society should be as well. With these assumptions, Harsanyi showed that if von Neumann–Morgenstern utility functions are used to represent the preferences, then alternatives are socially ranked according to a weighted sum of the individual utilities associated with them.

The interpretation of Harsanyi's impartial observer and social aggregation theorems as being theorems about utilitarianism has been controversial. In "Welfare Inequalities and Rawlsian Axiomatics" (1976), Amartya Sen (a 1998 Nobel laureate) argued that, contrary to what many believe, von Neumann–Morgenstern utility functions are not cardinal and, hence, cannot serve as a basis for a defense of utilitarianism. In "A Reconsideration of the Harsanyi-Sen Debate on Utilitarianism" (1991), John Weymark, while endorsing Sen's critique, showed how Harsanyi's utilitarian conclusions could be supported by incorporating ideas from Harsanyi's writings that are not stated explicitly in his theorems.

Harsanyi also wrote extensively about the philosophical issues related to his version of utilitarianism. He was a strong advocate for rule utilitarianism, the doctrine that utilitarian principles should be applied to rules for behavior, not individual acts.

Cooperative Games and Bargaining Theory. Game theory is concerned with the analysis of rational decision making by players (individuals or groups) when the outcome obtained by any player depends not only on the choices he makes, but also on the choices of the other players. In cooperative game theory, binding agreements are possible, whereas in noncooperative game theory, they are not.

In the 1950s, cooperative games dominated the research agenda of game theorists. In John Nash's 1950 article, "The Bargaining Problem," a two-player bargaining problem is described by the set of utility payoffs that are achievable for the players if they can reach an agreement and the payoffs that result if no agreement is reached (the threat point). A solution specifies the payoffs received by the players in each bargaining problem. Proceeding axiomatically, Nash identified a unique solution to all such problems. Earlier, Frederik Zeuthen, in his *Problems of Monopoly and Economic Warfare* (1930), had considered a dynamic approach to two-player bargaining in which, at each stage of the bargaining, the player who is less willing to risk a conflict makes the next concession. In "Approaches to the Bargaining Problem before and after the Theory of Games" (1956), Harsanyi showed that Bayesian decision makers would behave as Zeuthen suggested and that the final outcome of Zeuthen's bargaining process is the Nash solution.

In "A Value for *n*-Person Games" (1953), Lloyd Shapley axiomatically characterized a unique solution—the Shapley value—for any *n*-person transferable utility (TU) cooperative game. In a TU game, actions are available that permit the transfer of a unit of utility between any two players. In his Stanford PhD thesis, Harsanyi showed how to extend Shapley's solution to *n*-player cooperative games in which utility is not transferable. Furthermore, his general solution for cooperative games has Nash's bargaining solution for two-player games with variable threat points as a special case. Harsanyi's general solution for cooperative games is supported by a noncooperative threat game in which each coalition of individuals guarantees its members certain payoff levels by announcing a threat strategy that the coalition would implement if it cannot reach agreement with the coalition consisting of the rest of the players.

Games of Incomplete Information. By the early 1960s, Harsanyi had started shifting the focus of his research to noncooperative games. The extensive form of a noncooperative game specifies the order in which the players make decisions (simultaneous moves are not precluded), what actions are available and what information is known to a player about past choices each time he gets to make a decision, and the expected payoffs to each player at the end of the game as a function of the history of these decisions. Exogenous random events are modeled as decisions made by nature. In a game of complete information, the structure of the game is common knowledge, although at any time, players need not know the complete past history of play (in which case, the game is one of imperfect knowledge). A strategy for a player is a contingent plan of action that specifies what choice is to be made each time this player gets to make a decision. A mixed strategy includes nondeterministic choices, whereas a pure strategy does not. In the normal form of a game, the players are regarded as independently and simultaneously choosing these strategies once and for all at the beginning of the game. The decisions specified by these strategies are then implemented as the game unfolds. These strategies are a Nash equilibrium if no individual could change his strategy so as to achieve a higher payoff given the strategy choices of the other players.

The assumption that the payoffs obtained from each history of play is common knowledge in a game of complete information limits the applicability of this theory. In a game of incomplete information, players need not have full knowledge of the extensive form. In particular, a player need not know anyone else's payoff from a given history of play. However, prior to Harsanyi's pathbreaking three-part article, "Games with Incomplete Information Played by 'Bayesian' Players" (1967–1968), little progress had been made in analyzing games of incomplete information. Harsanyi's conceptual breakthrough was to recognize that it is possible to embed a game of incomplete information into a larger game of complete information and use it to determine equilibrium behavior in the original game. He did this by thinking of each player as potentially being one of a number of possible types, with each type corresponding to a different specification of a player's private information about the structure of the game, including this player's beliefs about the other players' types. The augmented game begins with a chance move by nature, made in accordance with a common prior probability distribution on the players' possible types, which determines the types that are to play the rest of the game. Following this chance move, each player learns his own type and updates his beliefs about the other players' types using Bayes's rule. At this point, the original incomplete information game begins. In this way, incomplete information about the other players' types in the original game is transformed into imperfect information about nature's initial decision in the augmented game, which is something that games of complete information were already equipped to handle.

A strategy for a player in the augmented game can be thought of as specifying a conditional plan of action for each possible type of this player. Viewed from this perspective, a Nash equilibrium can be equivalently described using Harsanyi's concept of a Bayesian-Nash equilibrium, which requires each type to choose a strategy so as to maximize its expected payoff, given the beliefs it has about the other players' types and given the strategies of the possible types of the other players. As Harsanyi recognized, this equilibrium concept is well defined even if the type-conditional beliefs are not derivable from a common prior. However, in a way reminiscent of his similarity principle, Harsanyi argued that differences in players' types can be accounted for by differences in their information and that prior to nature's initial move, everyone has the same information, so there should be a common prior. This argument is known as the Harsanyi doctrine.

From the time Harsanyi presented his research on games of incomplete information to the Jerusalem Game Theory workshop in 1965, it has had a major impact. For example, this research helped provide the theoretical basis for the Mathematica arms control project. Harsanyi's formalization of a game of incomplete information and his concept of a Bayesian-Nash equilibrium has become the standard way in which games of incomplete information are modeled and analyzed. His insights provided the foundation for much of the subsequent research on problems in which individuals are asymmetrically informed about economically relevant information.

Other Work on Game Theory. In the traditional interpretation of a mixed strategy in a game of complete information, a player chooses the probability that he wishes to assign to each of his pure strategies and then employs a random device to determine which of his pure strategies to implement. In a mixed-strategy Nash equilibrium, a player is indifferent between all of the pure strategies to which he assigns positive probability, but he randomizes so as to hide his intentions from the other players. However, the other players observe only the pure strategy that is actually implemented, which leads one to ask: Why randomize? In "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed Strategy Equilibrium Points" (1973), Harsanyi used his games of incomplete information to provide a reinterpretation of the meaning of a mixed strategy that resolves this paradox. Harsanyi supposed that a player's payoffs are subject to small random perturbations due to factors whose realization is known only to himself. The resulting game of incomplete information has a unique Bayesian-Nash equilibrium in which each type chooses a pure strategy. However, because a player only has probabilistic information about the types of the other players, it actually appears from the perspective of the first player that they are using mixed strategies even though they are behaving deterministically. By letting the size of the payoff perturbations go to zero, a mixed strategy equilibrium of the original game of complete information is obtained.

In "Two-Person Cooperative Games" (1953), John Nash had suggested that the binding agreements that are assumed to be possible in a cooperative game need to be justified by showing that they can arise as equilibrium outcomes in some noncooperative game. The search for noncooperative foundations for cooperative games is known as the Nash program. The noncooperative elements of Harsanyi's general solution for cooperative games can now be seen to be a step toward Harsanyi's full-fledged support of the Nash program. He made a major contribution to this program in "An Equilibrium-Point Interpretation of Stable Sets and a Proposed Alternative Definition" (1974) by providing a noncooperative foundation for the solution for cooperative games proposed by von Neumann and Morgenstern in their *Theory of Games and Economic Behavior.*

A major theme of Harsanyi's work on game theory is that the goal of game theory should be to use Bayesian principles of rationality to determine a unique solution to any game. Games often have multiple equilibria, so in order to achieve this goal, some procedure must be used to select among the equilibria. This research agenda reached its apogee in Harsanyi's *A General Theory of Equilibrium Selection in Games* (1988), with Reinhard Selten, in which the selection is accomplished using an approach in which the tracing procedure introduced by Harsanyi in "The Tracing Procedure: A Bayesian Approach to Defining a Solution for n-Person Noncooperative Games" (1975) plays a major role.

The tracing procedure identifies a unique equilibrium in a noncooperative game by analyzing equilibrium behavior in a continuum of auxiliary games that differ from the original game only in the payoffs players receive from the possible strategy combinations. This procedure begins with an auxiliary game in which a probability distribution over a player's pure strategies is given a priori. This distribution represents the initial conjecture on the part of the other players about this player's mixed strategy choice. The payoff to any player from a strategy choice in this auxiliary game is the payoff that would be obtained in the original game if the other players played according to the initially conjectured strategies. In this game, each player has a unique best response to the conjectured strategy choices of the other players, but these best responses are typically not a Nash equilibrium in the original game. Next, for each number t between 0 and 1, a t-auxiliary game is defined in which the payoffs to players are weighted combinations of the payoffs they would obtain in the original game and the initial auxiliary game, with weights t and $1 - t$, respectively, plus a small additional payoff that ensures that the equilibrium in each of the t-auxiliary games is unique. The value $1 - t$ represents the degree of confidence placed in the initial conjecture. The equilibria defined by this procedure converge to a unique equilibrium in the 1-auxiliary game, which is a unique equilibrium in the original game when the values of the small added payoffs go to zero. Harsanyi interpreted the tracing procedure as being a mathematical formalization of the process by which rational players coordinate their choices of strategies.

Harsanyi continued to work on equilibrium selection until his final illness ended his research career. In his 1995 articles on this topic, "A New Theory of Equilibrium Selection for Games with Complete Information" and "A New Theory of Equilibrium Selection for Games with Incomplete Information," Harsanyi's tracing procedure, which for two decades had been an important component of the Harsanyi-Selten theory of equilibrium selection, plays only a minor role.

There is a unity in Harsanyi's research that is quite remarkable when one considers the range of problems that he considered over his lifetime. In his 1977 monograph, *Rational Behavior and Bargaining Equilibrium in Games and Social Situations,* Harsanyi announced that his goal was to provide a systematic account of rational behavior based on Bayesian principles that yields determinate solutions in individual decision making, in games, and in moral decision making. In retrospect, one can see that most of what Harsanyi wrote contributed to the achievement of this objective.

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John A. Weymark

HARTREE, DOUGLAS RAYNER

(*b.* Cambridge, England, 27 March 1897, *d.* 12 February 1958), *mathematics, theoretical physics, quantum chemistry, computing, numerical analysis.*

Hartree played a fundamental role in the field of twentieth-century numerical analysis and its application to theoretical physics. He developed practical numerical methods for use with pen and paper, desk calculating machines, differential analyzers, and electronic computers, and he pioneered the application of calculating technologies to scientific problems. In mathematical physics Hartree's most well-known contribution was the invention of the method of the self-consistent field for calculating atomic wave functions, which became known as the Hartree-Fock approximation, following further work on the technique by Vladimir Fock. This and other contributions meant that during the 1920s and 1930s, Hartree played an important role in the development of atomic physics and quantum chemistry, work for which he was elected a Fellow of the Royal Society in 1932.

Hartree specialized in the numerical solution of ordinary and partial differential equations—equations that often described real world problems and therefore needed real world solutions. From his early work on ballistics through research on quantum chemistry, Hartree used the latest computing technology to find practical solutions to differential equations. He was responsible for bringing Vannevar Bush's differential analyzer technology to the United Kingdom and for developing a wide range of scientific and industrial applications for the machine. In the post–World War II period, Hartree was influential in gaining support for the development of electronic computers in England and devising numerical methods for their application to problems in theoretical physics. One of his final contributions was the book *Numerical Analysis,* first published in 1952 and regarded as a classic in the subject.

Origins and Early Career. Hartree was born in Cambridge, England, in 1897. His father, William Hartree, taught in the Engineering Laboratory at Cambridge University until his retirement in 1913 at the age of forty-three. William Hartree was very skilled in numerical computation and continued to undertake scientific work after his retirement from Cambridge, as an assistant to both A. V. Hill and, later, to his son. Hartree's mother, Eva Raynor, was very active in public affairs, working with the Red Cross, the suffragette movement, the League of Nations Union, and the British National Council of Women. She served on the Cambridge Borough Council for twenty years and was the first female mayor of Cambridge in 1925.

Hartree was educated first at a small school in Cambridge and then at Bedales School in Petersfield in Hampshire, from which he won a scholarship to study mathematics at the University of Cambridge in 1915. Hartree completed one year of his undergraduate degree before leaving Cambridge to undertake war work with the Ministry of Munitions. The main role of the Ministry of Munitions was to supply the British Forces with weapons and ammunition throughout World War I. Hartree was invited to join A. V. Hill's Anti-Aircraft Experimental Section of the Munitions Inventions Department of the Ministry of Munitions as a commissioned lieutenant in the Royal Naval Volunteer Reserve, as part of a team made up largely of Cambridge mathematicians and mathematical physicists, including Ralph Fowler, Edward Milne, and Hartree's father. William Hill, a Cambridge physiologist and later pioneer of operations research, had been charged by the Ministry of Munitions with undertaking ballistics research to assist in the development of new anti-aircraft weapons.

The work was a mix of routine ballistics calculations and mathematical research on the ballistics of high-angled fire. Hartree became expert at both pencil and paper calculations and the use of hand-cranked calculating machines, such as the Brunsviga, but he also began to develop new numerical processes to calculate trajectories. His most lasting innovation was the use of time rather than angle of elevation as the independent variable in trajectory calculations, but it was his development and refinement of practical iterative methods for the numerical solution of differential equations that was to shape his future career. After the war Hartree wrote up his work on ballistics calculations for the journal *Nature* (1920) and coauthored a paper with Leonard Bairstow and Ralph Fowler on the pressure distribution on the head of a shell traveling at high velocities, published in the prestigious *Proceedings of the Royal Society,* thereby signaling the start of his career as a mathematical physicist.